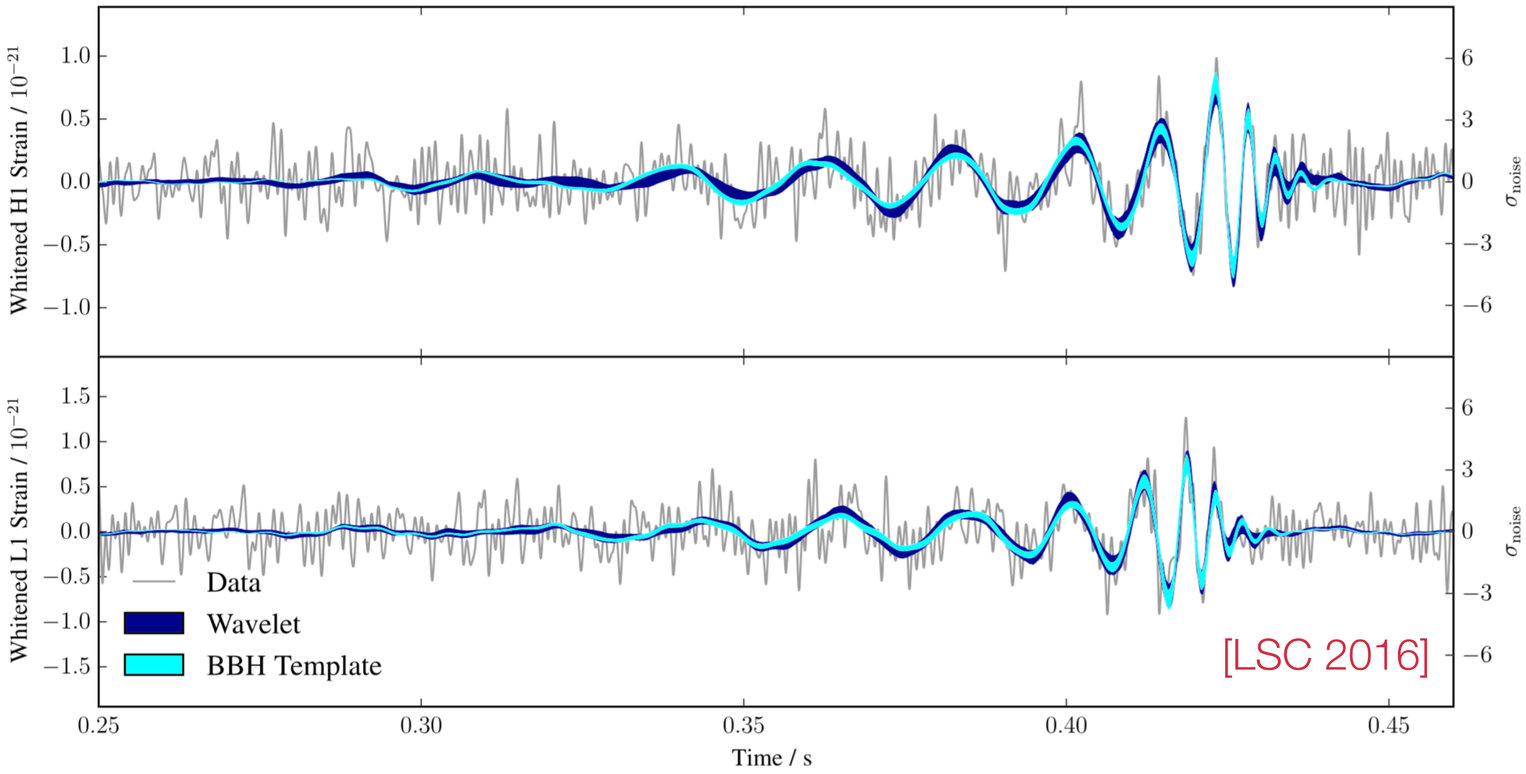
Testing general relativity with black-hole binary observations: a manifesto

Michele Vallisneri
Jet Propulsion Laboratory, Caltech

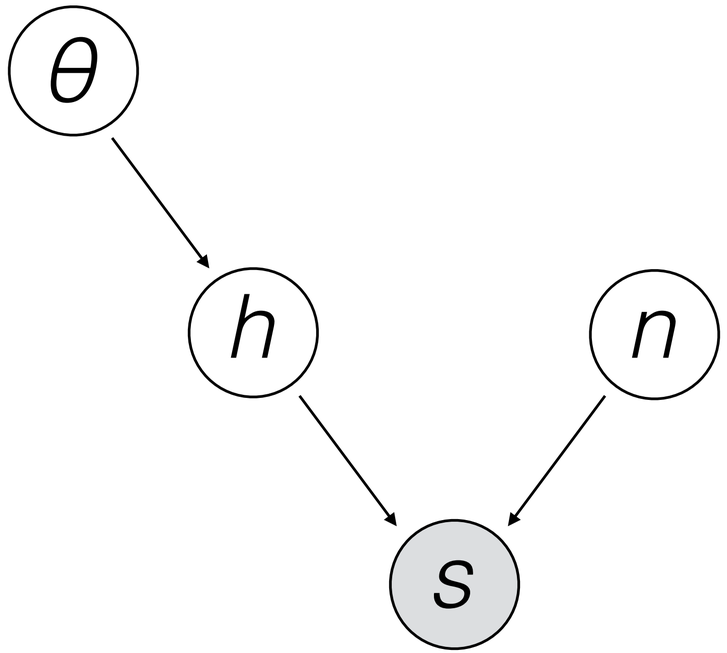


- Detecting GWs offered a profound confirmation of general relativity
- Using GW observations to test GR to increasing precision, we hope to obtain clues of new physics, and ultimately prove a more general theory

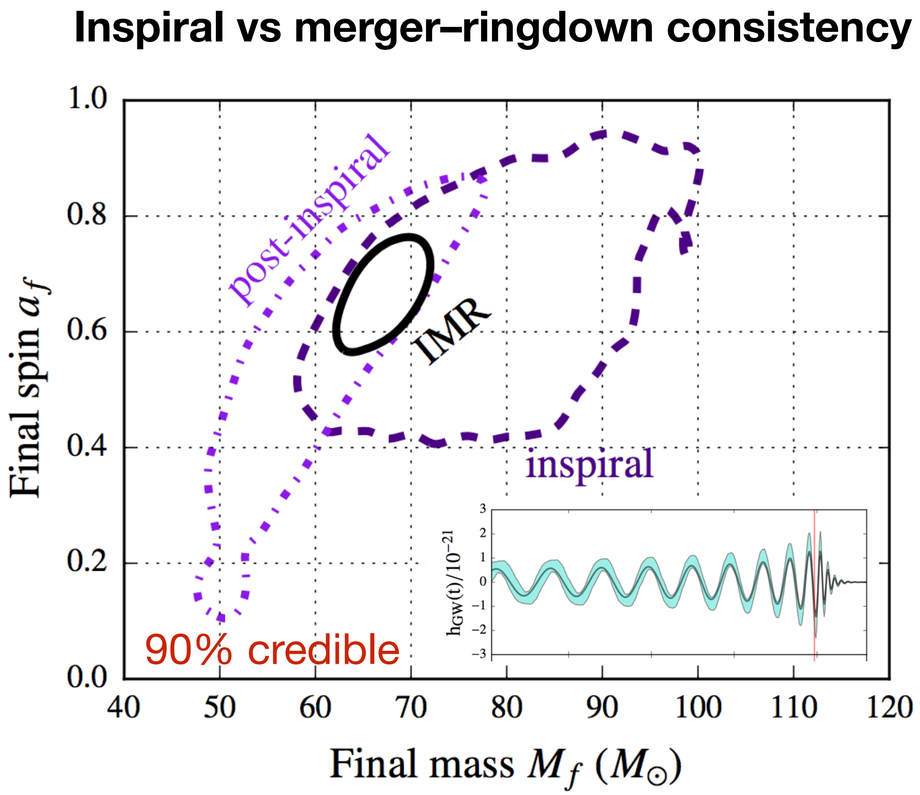
Testing GR with GW observations: a manifesto

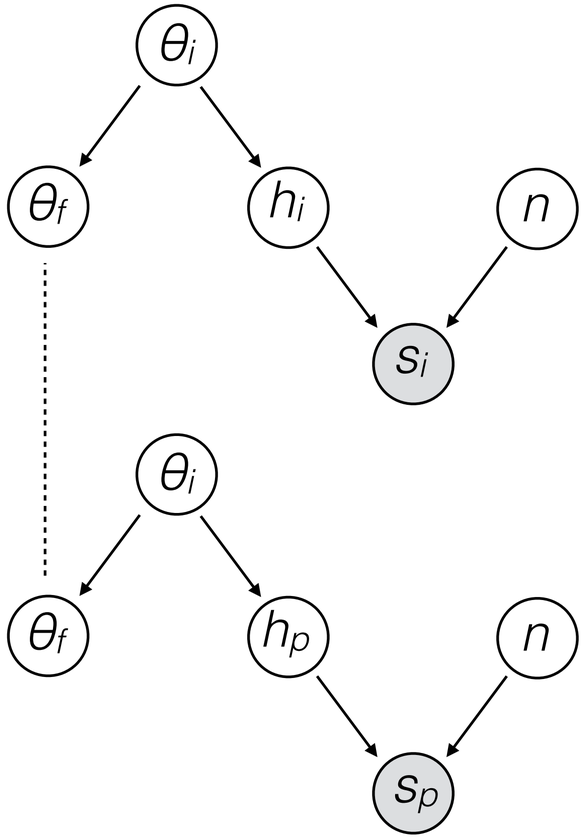
- **Consistency**: a useful sanity check, but hard to interpret statistically. P values are possible with much work. But would we ever believe an inconsistent result?

the graphical model of parameter inference



GW150914 tests of “consistency”: partial waveforms (1)





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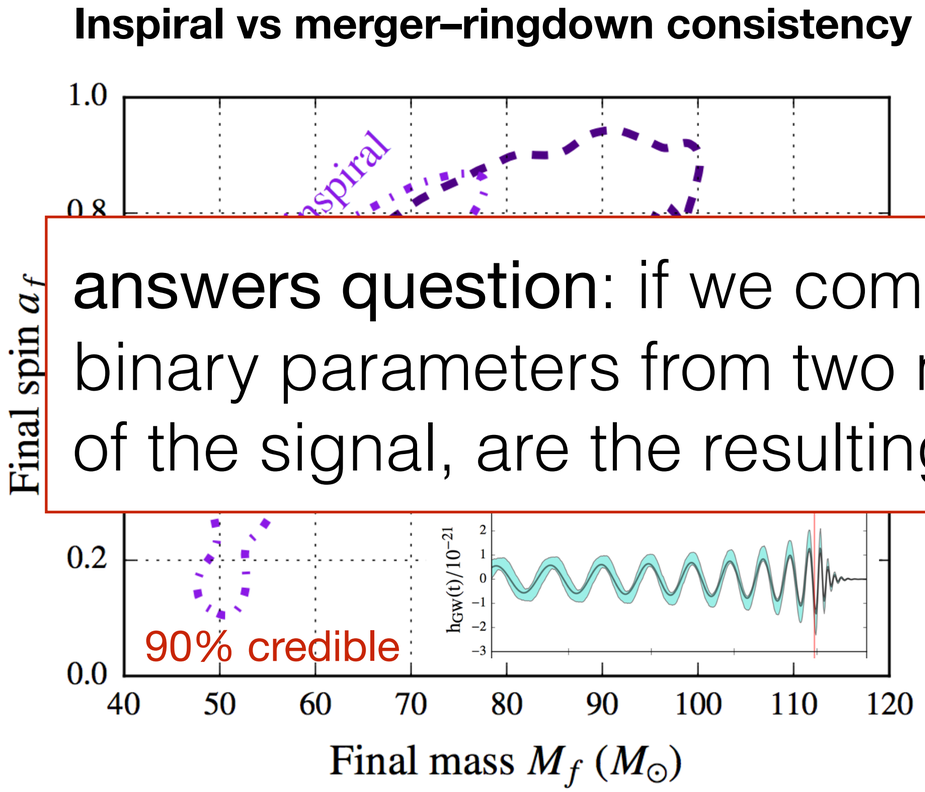
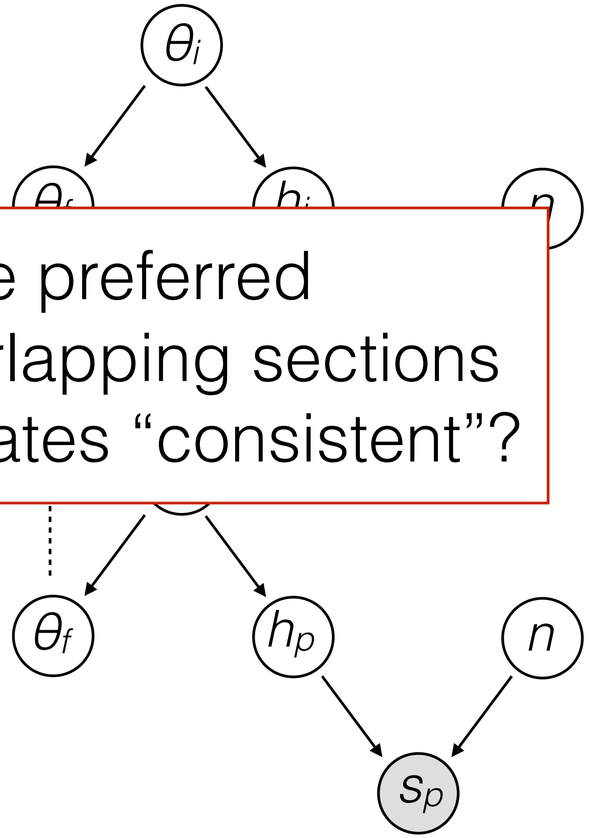
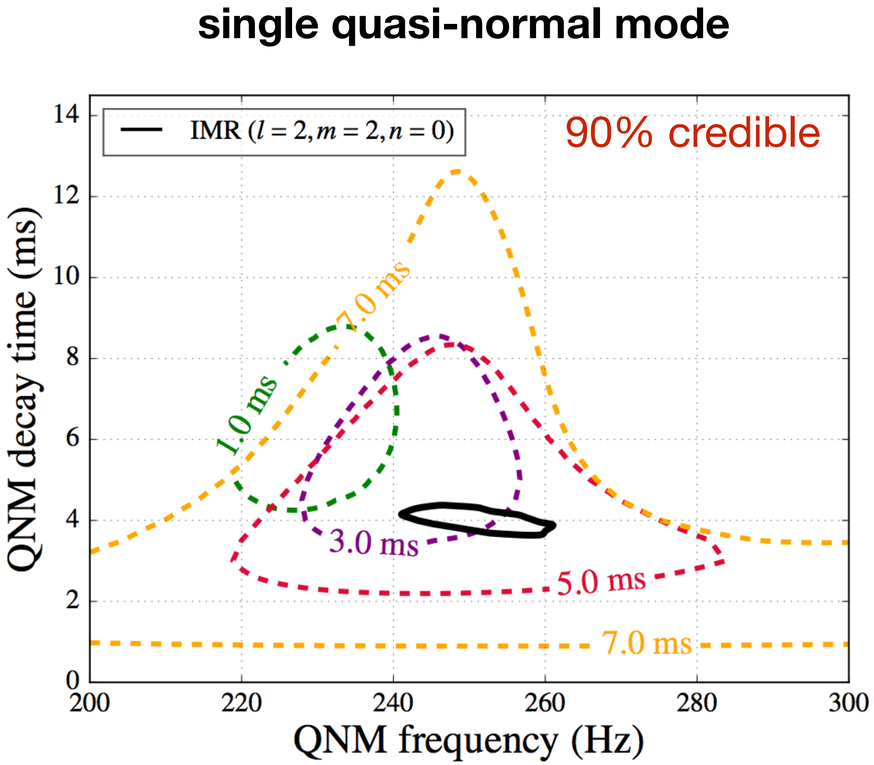
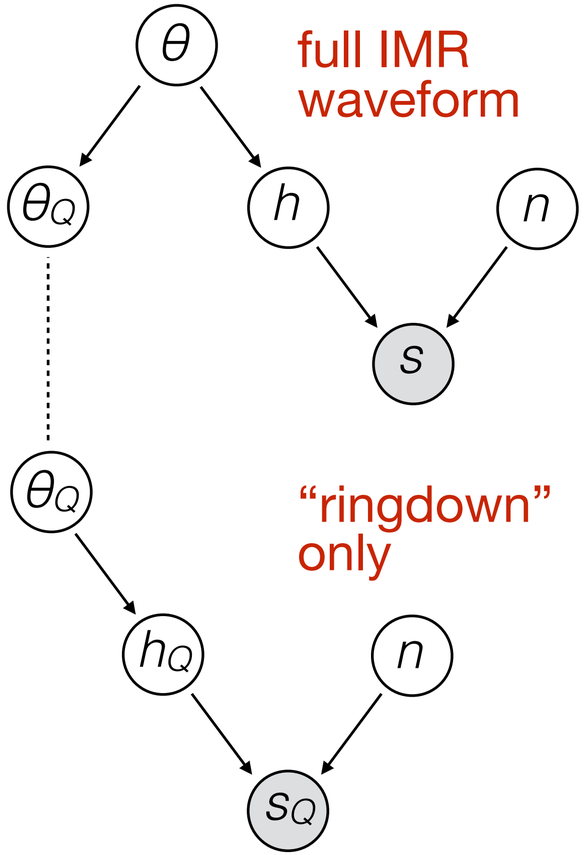


answers question: if we compute the preferred binary parameters from two nonoverlapping sections of the signal, are the resulting estimates “consistent”?



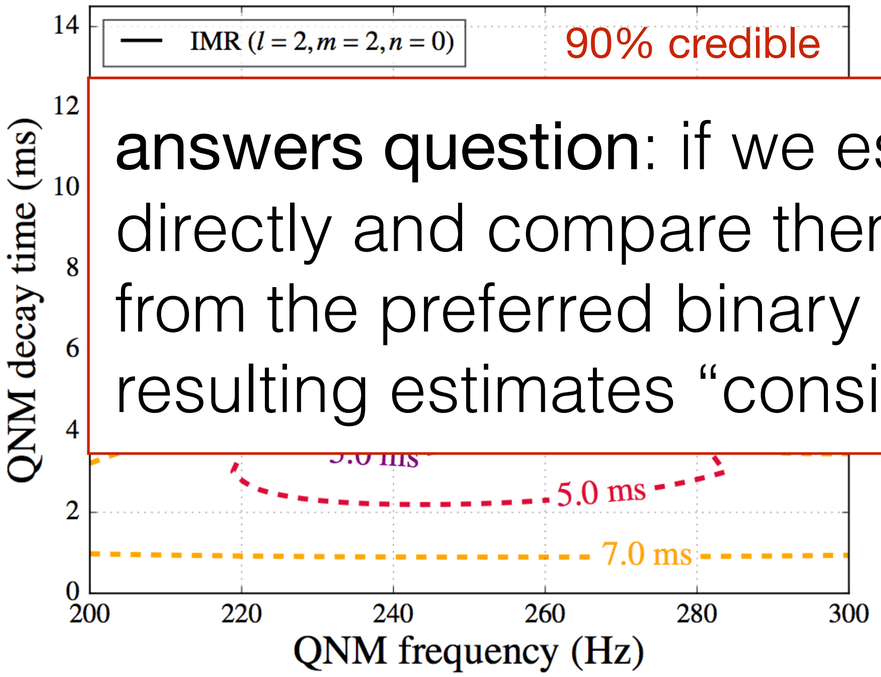
GW150914 tests of “consistency”: partial waveforms (2)



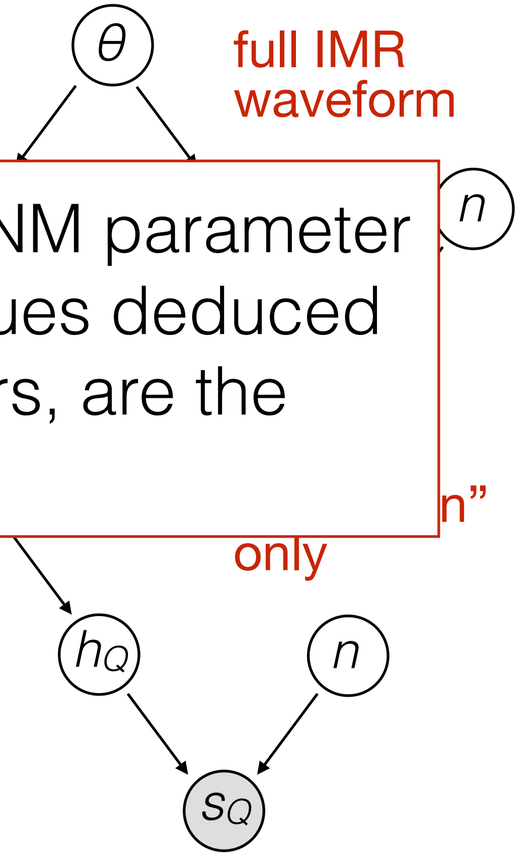


GW150914 tests of “consistency”: partial waveforms (2)

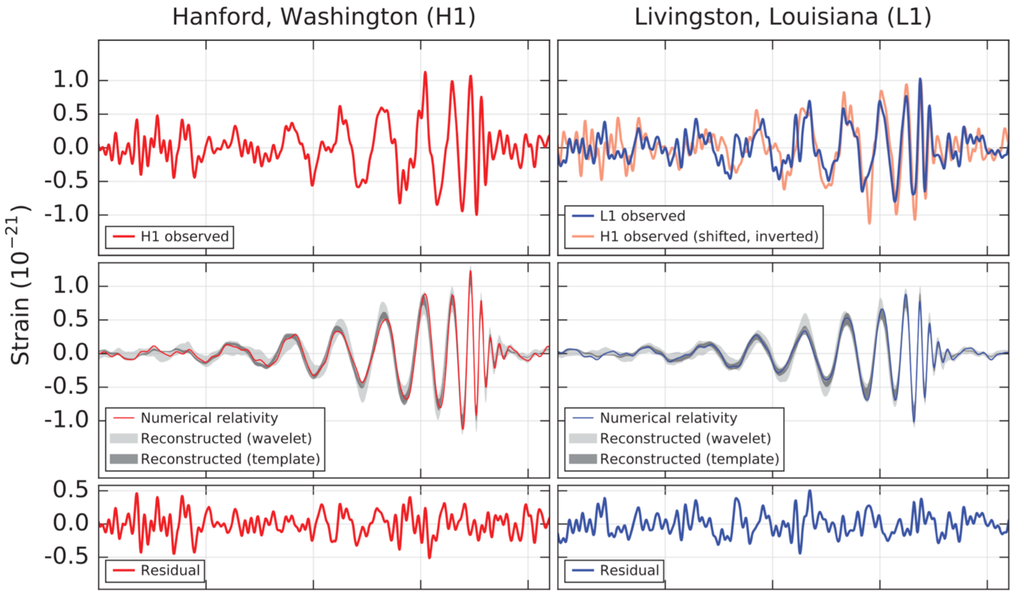
single quasi-normal mode

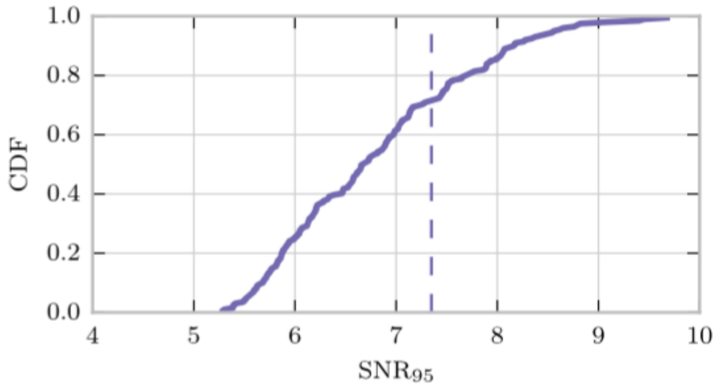


answers question: if we estimate QNM parameter directly and compare them with values deduced from the preferred binary parameters, are the resulting estimates “consistent”?



GW150914 test of “consistency”: full-waveform residual





SNR in coherent burst analysis of data residual after subtracting best-fit GW150914 waveform

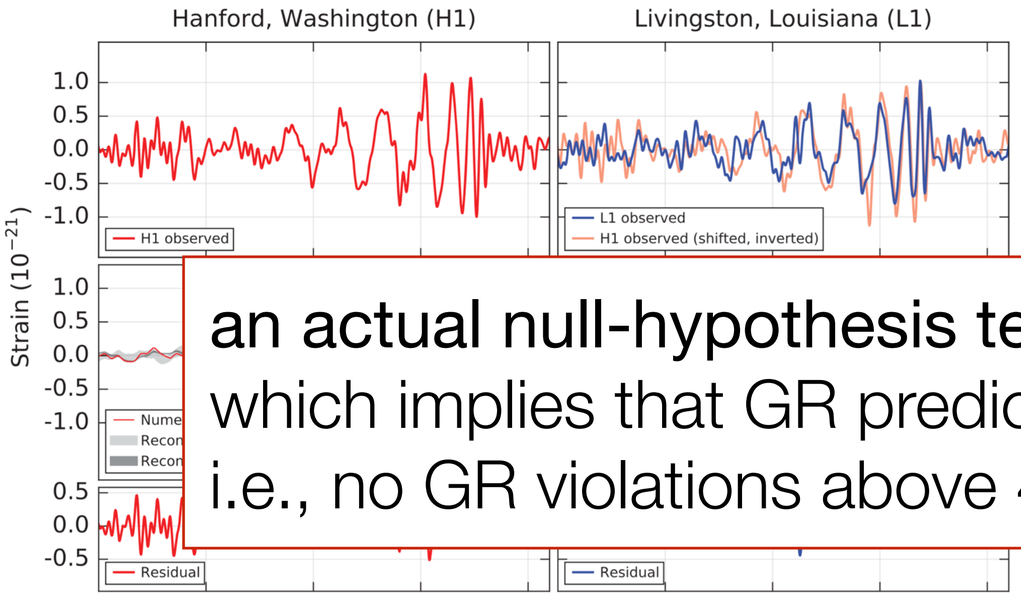
$$\mathrm{SNR}^2_{\mathrm{res}} = \frac{1-\mathrm{FF}^2}{\mathrm{FF}^2}\mathrm{SNR}^2_{\mathrm{det}}$$

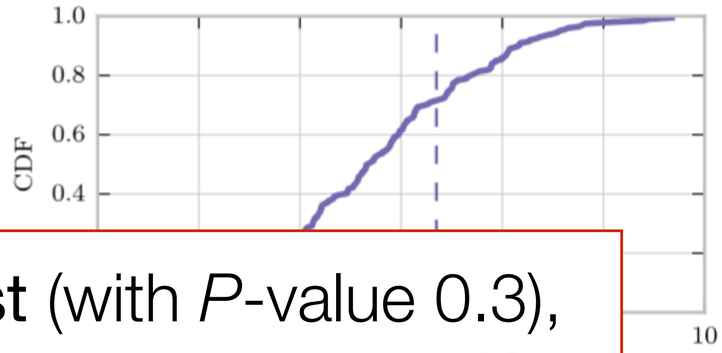
Fitting Factor: parameter-maximized waveform overlap

$$\mathrm{SNR}_{\mathrm{res}} \leq 7.3 \Rightarrow \mathrm{FF} \geq 0.96$$

(for violations not absorbed by physical parameters)

GW150914 test of “consistency”: full-waveform residual





an actual null-hypothesis test (with *P*-value 0.3), which implies that GR prediction is verified to 4%; i.e., no GR violations above 4% of waveform

best-fit GW150914 waveform

$$\mathrm{SNR}^2_{\mathrm{res}} = \frac{1-\mathrm{FF}^2}{\mathrm{FF}^2}\mathrm{SNR}^2_{\mathrm{det}}$$

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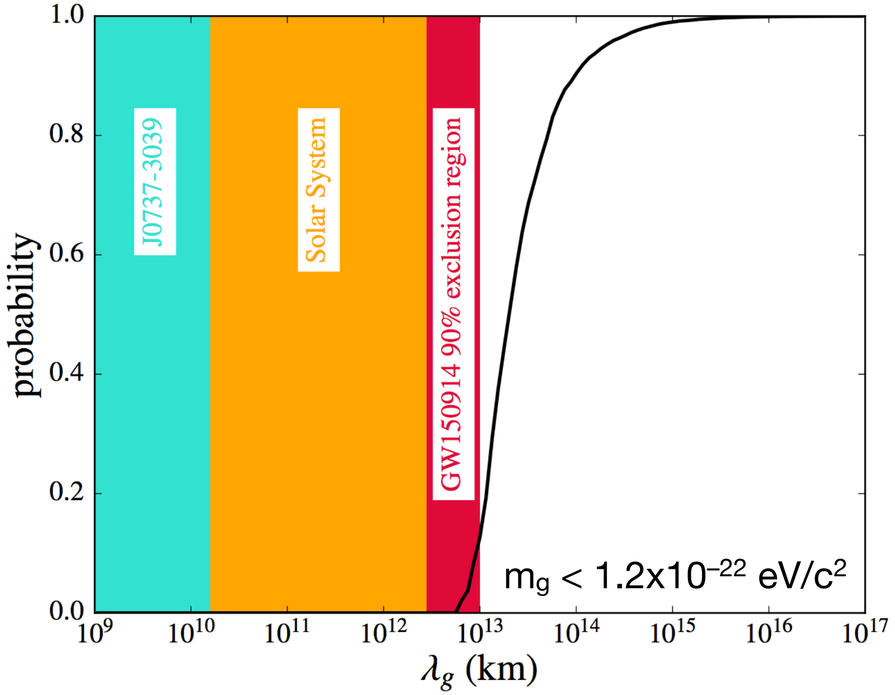
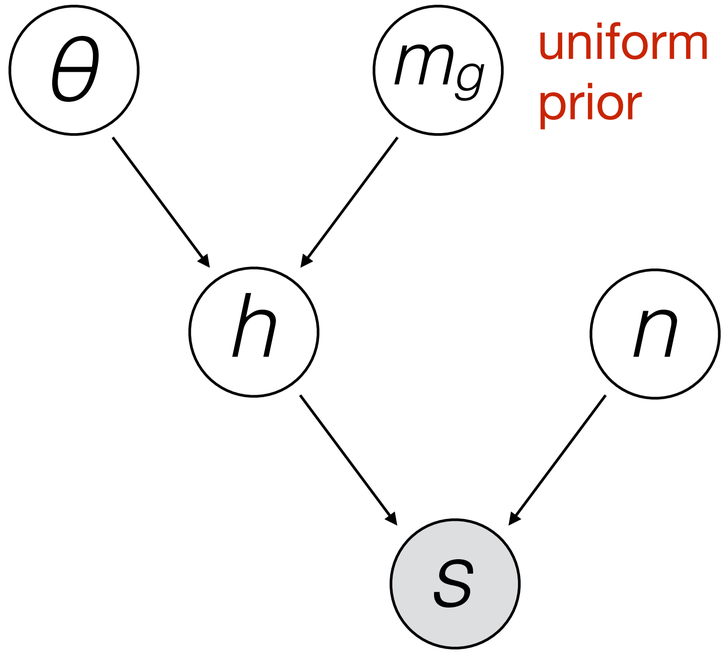
GW150914 parametric test of GW propagation: graviton mass





$$h(f) = \frac{1}{D}\frac{\mathcal{A}}{\sqrt{\dot{F}}} f^{2/3} e^{i\Psi(f)}$$

$$\Psi(f) = \sum_i \left[\psi_i + \psi_{il}\log f\right] f^{(i-5)/3} + \Phi^{\mathrm{MR}}[\beta_i, \alpha_i]$$

$$\frac{v_{\mathrm{g}}^2}{c^2} = 1 - \frac{m_{\mathrm{g}}^2 c^4}{E^2}$$

$$\delta\Psi(f) = \frac{\pi D c}{\lambda_g^2 (1+z) f}$$

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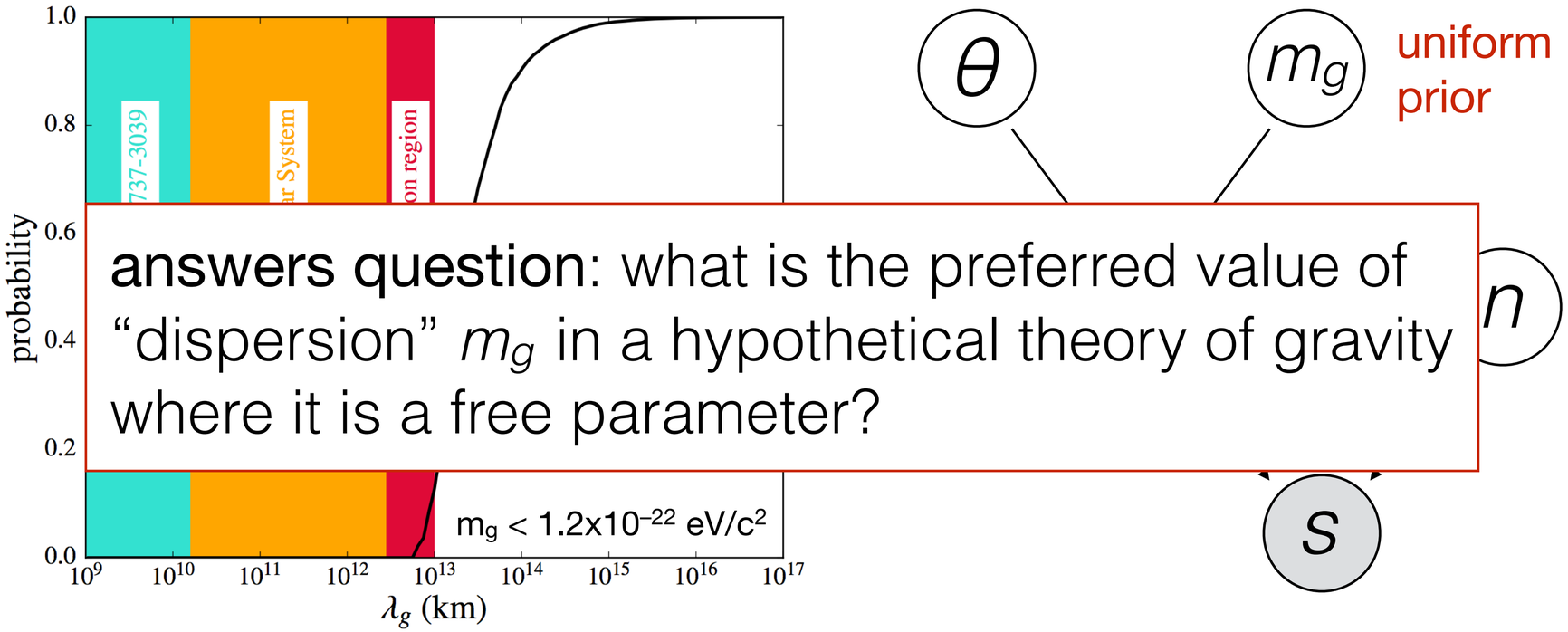

answers question: what is the preferred value of "dispersion" m_g in a hypothetical theory of gravity where it is a free parameter?

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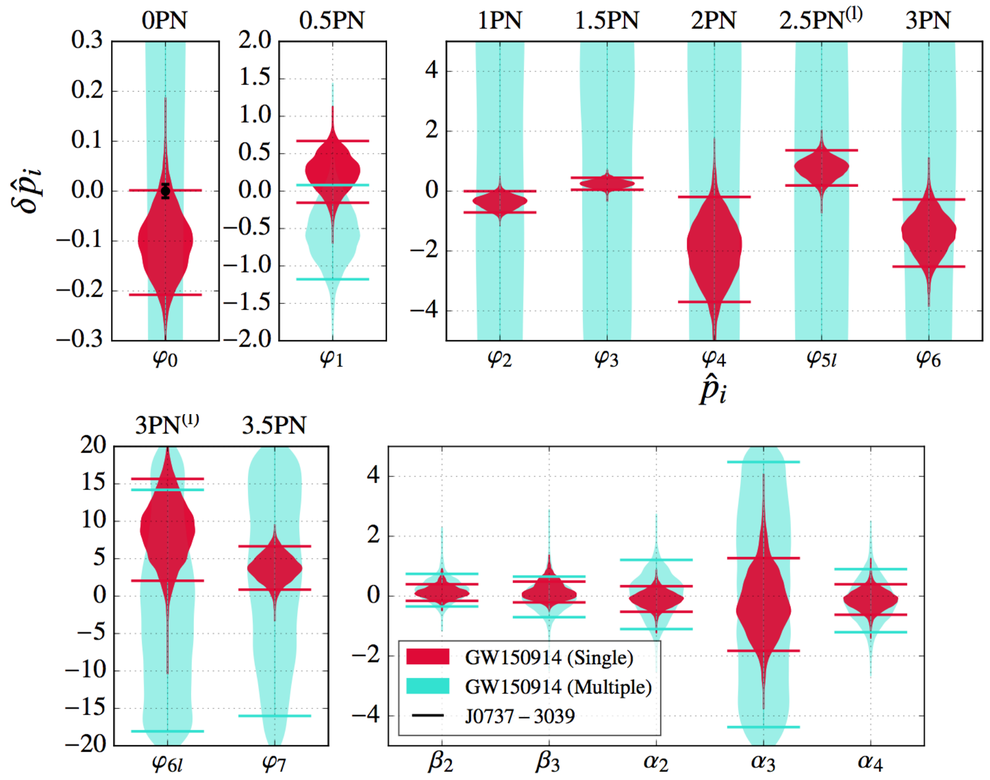
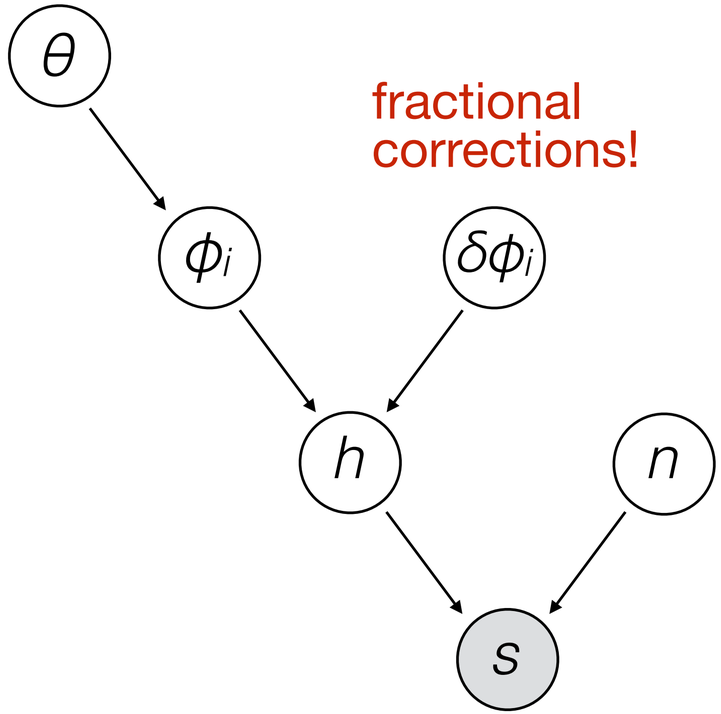
GW150914 test of generation and binary dynamics: frequency-domain PN coefficients





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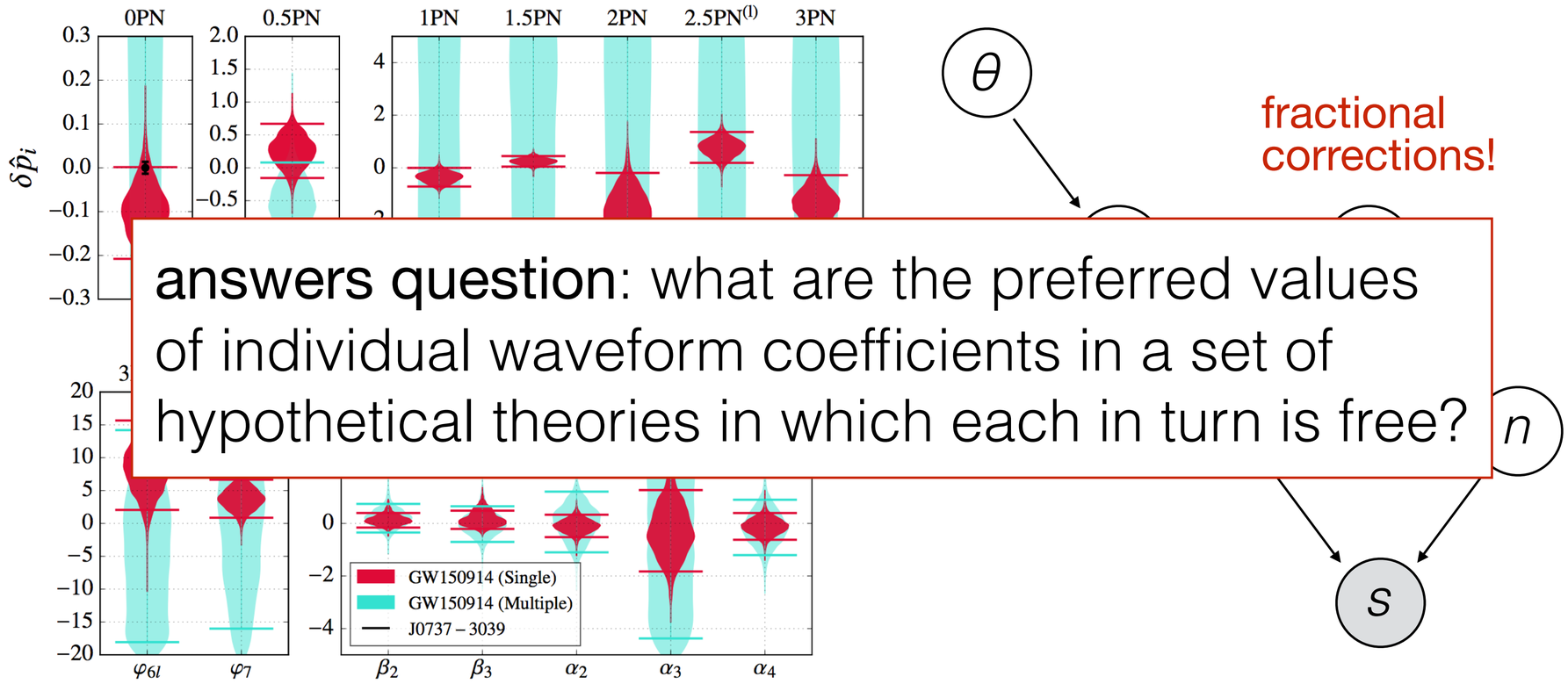

fractional corrections!

answers question: what are the preferred values of individual waveform coefficients in a set of hypothetical theories in which each in turn is free?

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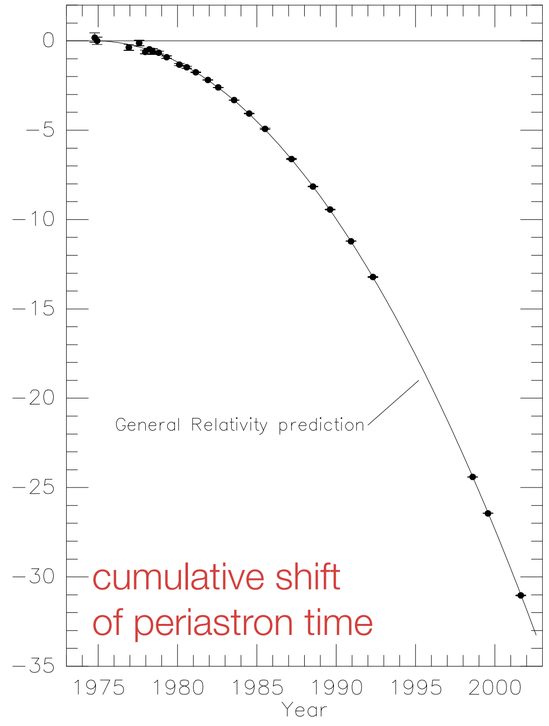
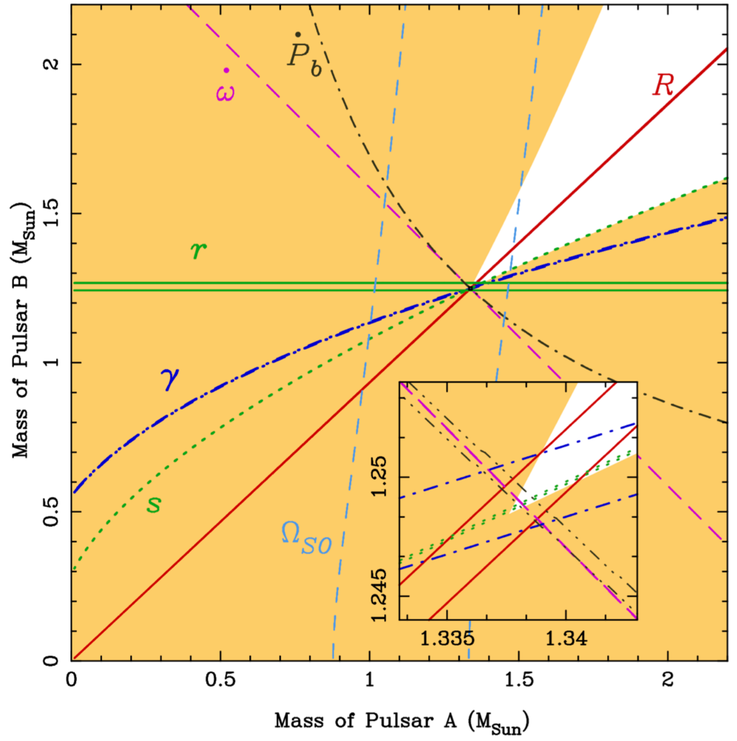
For comparison: pulsar-timing tests of GR concern physical parameters, but also have weak interpretations

PSR B1913+16
[Weisberg 2003]





PSR J0737-3039
[Kramer 2016]

$$\dot{P}_{\rm b} = -\frac{192\pi}{5}\left(\frac{P_{\rm b}}{2\pi}\right)^{-5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2}\,T_\odot^{5/3}\,m_1 m_2\,M^{-1/3}$$

$$\dot{\omega} = 3\left(\frac{P_{\rm b}}{2\pi}\right)^{-5/3}(T_\odot M)^{2/3}\,(1-e^2)^{-1}, \qquad r = T_\odot\, m_2,$$

$$\gamma = e\left(\frac{P_{\rm b}}{2\pi}\right)^{1/3} T_\odot^{2/3} M^{-4/3}\, m_2(m_1+2m_2), \quad s = x\left(\frac{P_{\rm b}}{2\pi}\right)^{-2/3} T_\odot^{-1/3}\, M^{2/3}\, m_2^{-1}.$$

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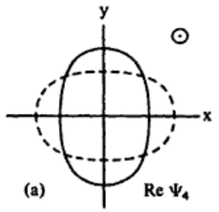
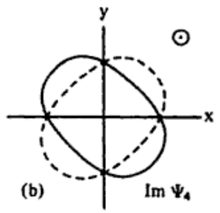
same as detection

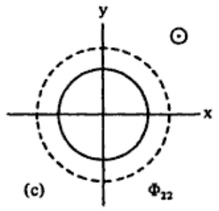
other measurements, particle physics, astroparticles

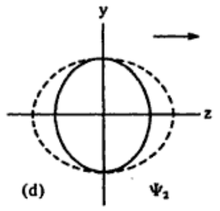
GW150914 test of polarization



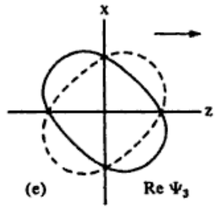


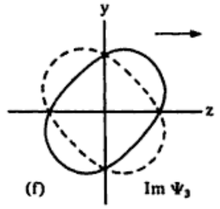
Tensor



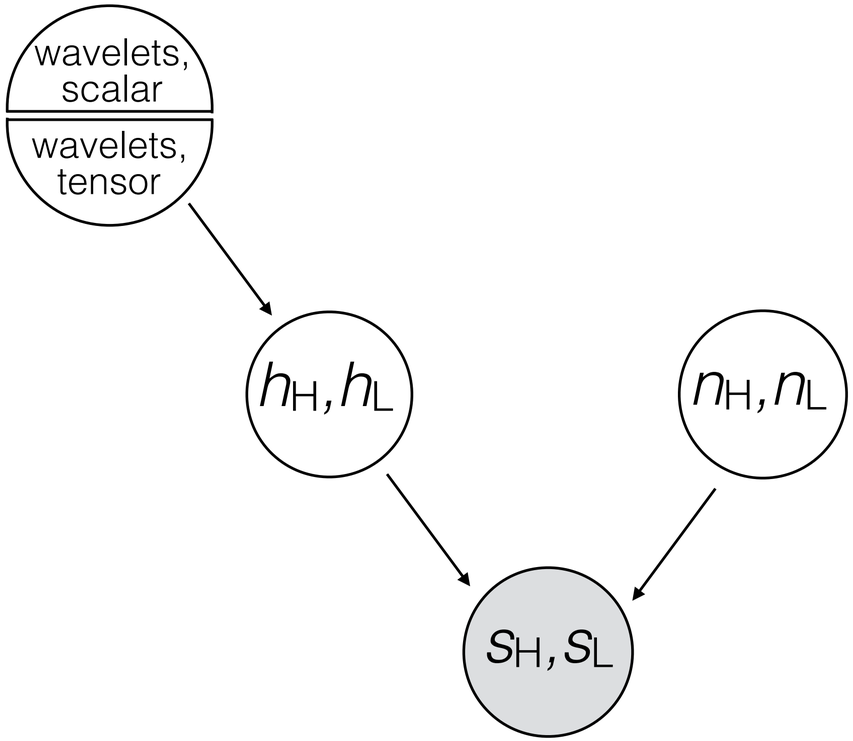


Scalar





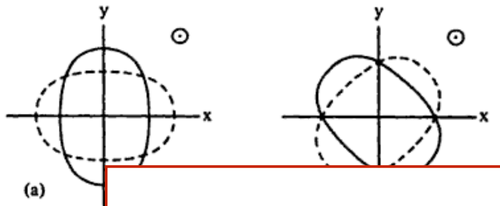
Vector



no discrimination between S and T

$$\frac{P(s|\mathrm{S})}{P(s|\mathrm{T})} \sim 1$$

GW150914 test of polarization

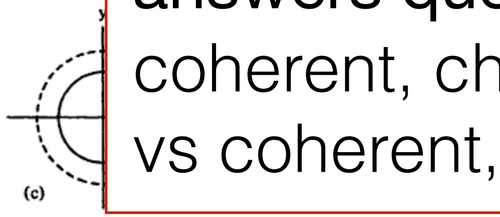


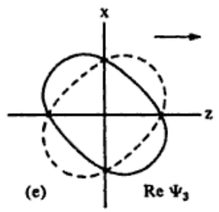
Tensor

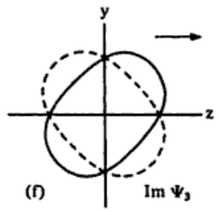
wavelets, scalar

wavelets, tensor

answers question: is there evidence for coherent, chirp-like quadrupolar radiation vs coherent, chirp-like purely scalar radiation?







Vector

n_H, n_L

s_H, s_L

no discrimination between S and T

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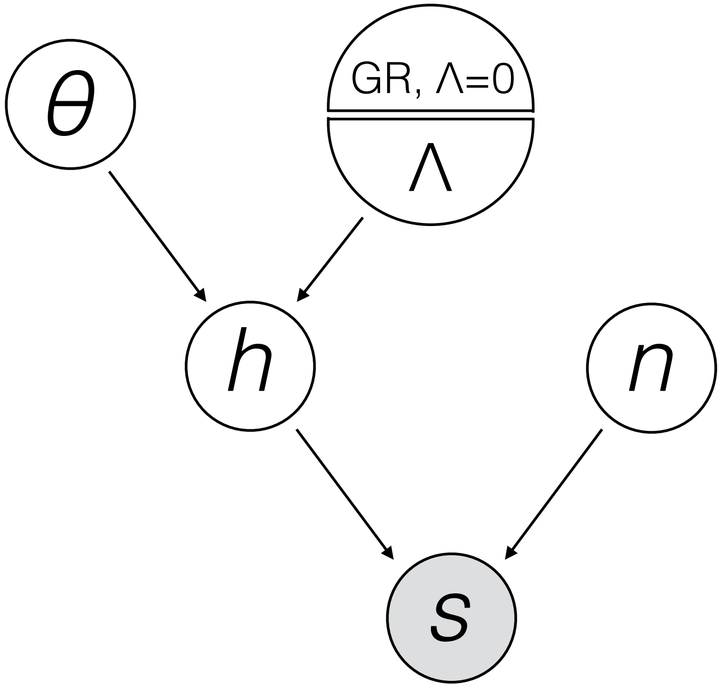
Establishing alternative theories



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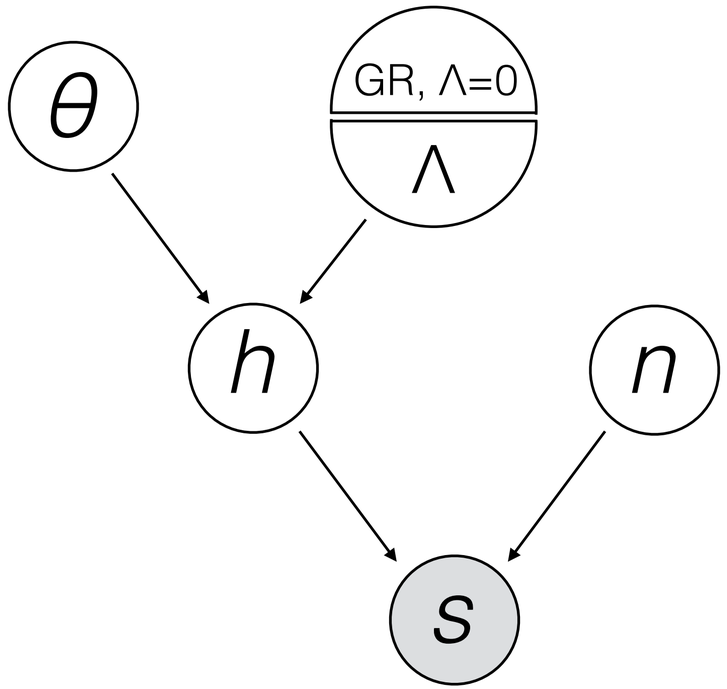

new physics follows from establishing an **anomaly**: we need to obtain convincing evidence that the data prefers an alternative theory of gravity over GR

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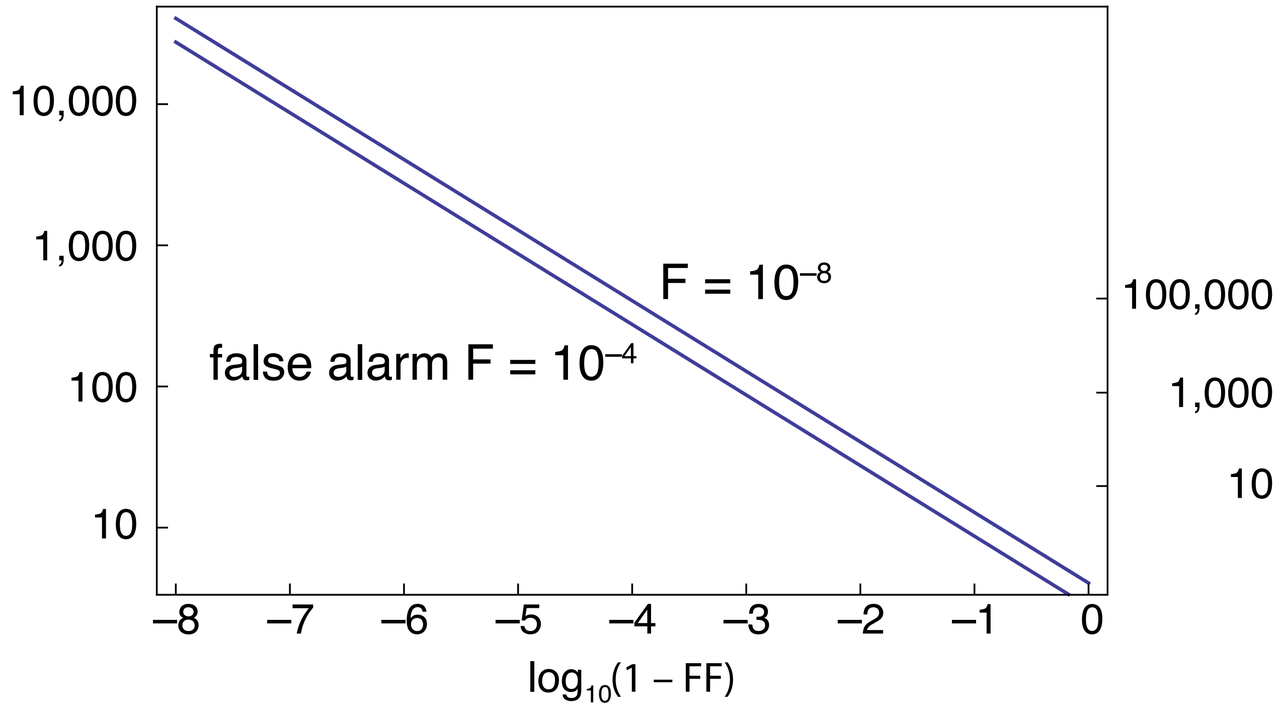
- **Size of effects**: detection SNR determines the magnitude of detectable waveform anomalies. 1% LIGO, up to 10^{-5} for LISA and future ground-based detectors.

How well can we hope to do? [MV, PRD 86, 2012]

- For a fixed false-alarm rate, we ask what **SNR** is needed to detect AG with 50% probability as a function of **fitting factor FF**, using the Bayesian odds ratio as “detection” statistic.

SNR to detect AG



events to yield SNR

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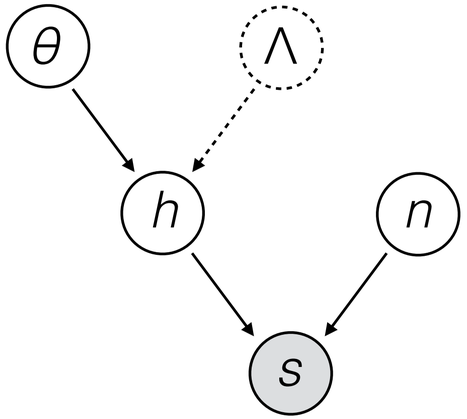
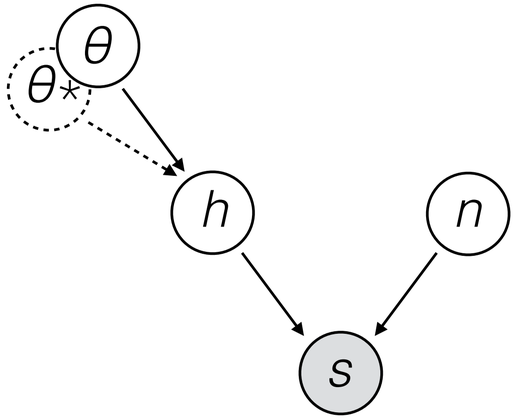
Control of systematics

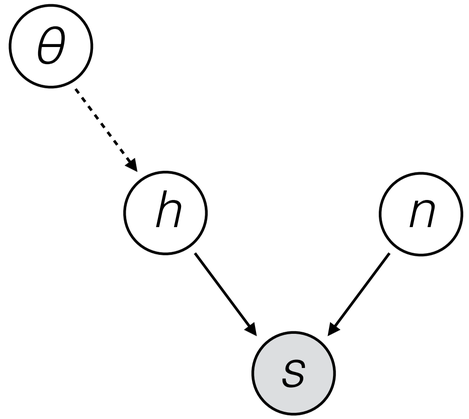
fundamental bias (Yunes)



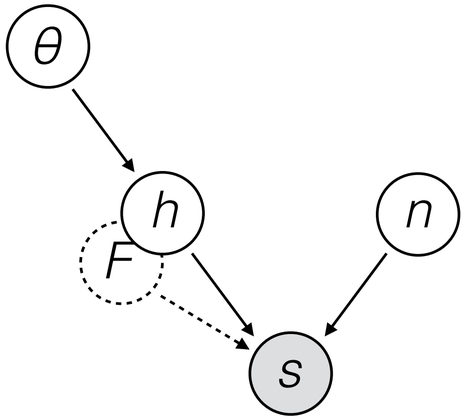
astrophysical bias



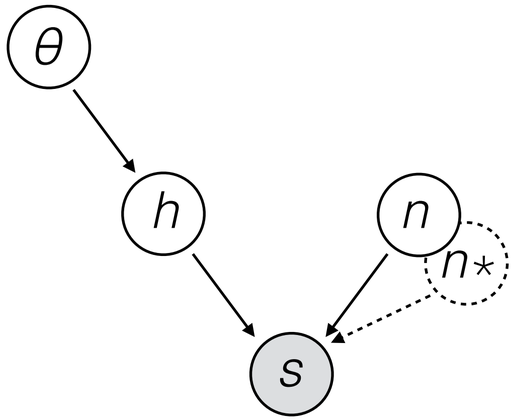
mismodeling



calibration



glitches



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develop!

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